JHMC Practice Questions

7^{th} Grade Individual

1. A farmer has to fence a regular hexagonal field with side length 6 feet. How many feet of fence does he need?

2. What is the value of $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10 + 11 - 12$?

3. Which has a greater area: a square with side length 3 or a triangle with side lengths 3, 4, and 5?

4. Out of 25 people in an IMSA math class, 18 know how to find the variation of a set of data, 13 know how to find the standard deviation, and 4 don't know how to find either of them. How many students know how to find both?

5. Two cubes have edge lengths of 6 inches and 12 inches. What is the ratio of the surface area of the smaller cube to the surface area of the larger cube? Express your answer as a common fraction.

6. Suppose x, y, and z are real numbers such that

$$\begin{aligned} x + 2y + 3z &= 9 \\ 4x + 3y + z &= 24 \\ 2x + 2y + 3z &= 30 \end{aligned}$$

   Find $x + y + z$.

7. What positive integer is 56 less than its square?

8. There are 12 eggs in a carton, 8 cartons in a crate, and 9 crates in a container. How many eggs are in a container?

9. The sum of the squares of two positive integers is 193. The product of the two integers is 84. What is the sum of the two integers?

10. What is the sum of all the distinct positive two-digit factors of 144?

8^{th} Grade Individual

1. A cafeteria serves 5 different drinks, 6 different appetizers, and 3 different main courses. How many different meals can you get (1 drink, 1 appetizer, and 1 main course)?

2. A rope of length 90 feet is cut into three pieces with lengths in a ratio of $1 : 2 : 3$. What is the length of the longest piece in feet?

3. Solve for x: $\sqrt{2x+3} = x + 2$.

4. The supplement of an angle is equal to six times its complement. What is the measure of the angle (in degrees)?

5. A combination lock uses three integers in the combination, and the dial is numbered with the integers 0, 1, 2, and 3. If consecutive numbers in the combination cannot be the same, how many possible combinations are there?

6. Four consecutive positive integers have a product of 840. What is the largest of the four integers?

7. Suppose 64 unit cubes are placed together to create a large cube. How many cubes with integer dimensions are in the $4 \times 4 \times 4$ cube?

8. A pentagon has m diagonals (not including sides) and a heptagon has n diagonals. Find $m - n$.

9. In year N, the 300^{th} day of the year is a Tuesday. In year $N + 1$, the 200^{th} day is also a Tuesday. On what day of the week did the 100^{th} day of year $N - 1$ occur?

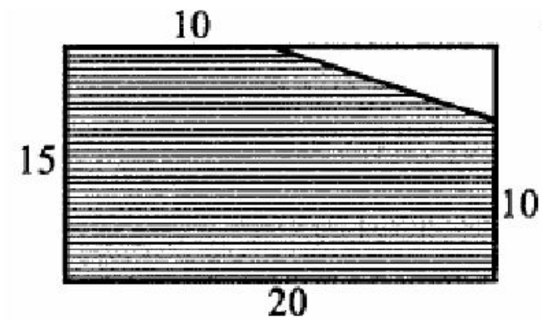
10. Suppose that $40!$ has m divisors and $41!$ has n divisors. Find $\frac{m}{n}$.

7^{th} Grade Team

1. The probability that it will snow tomorrow is 7/10. What is the probability that it will not snow tomorrow?

2. In triangle ABC, side AB is 5 units long, side BC is 8 units long, and the altitude to side BC is 3 units long. What is the area of the triangle?

3. A triangle is removed from the corner of a rectangle as shown. What is the area of the shaded pentagon?



4. Each digit in the number 2005 is placed on a different card. In how many ways can three different cards be selected so that the product of the numbers on those cards is not zero?

5. How many diagonals (not including sides) are in a regular heptagon?

6. Seven students ordered \$15.00 worth of pizza. They split the cost as evenly as possible, so some students paid \$2.14 each, and the others paid \$2.15 each. How many students paid the lesser amount?

7. Find the sum of the positive integer factors of 432 (including itself).

8. What is the area of a triangle with sides $13, 14$, and 15?

9. The number $M = 999 \cdots 999$ consists of 2005 9's. Find the sum of the digits of M^2.

10. One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

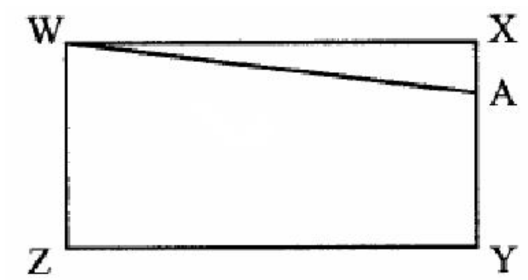
8^{th} Grade Team

1. Elliot has five different Beatles albums. The first has 10 songs, the second has 11, and the other three have 12. How many Beatles songs does Elliot have in total?

2. The integer x has 12 positive integer factors. If 12 and 15 are factors of x, what is x?

3. What is the least positive integer n such that $n^2 - n$ is divisible by some but not all of the positive integers less than or equal to n.

4. If WA divides rectangle $WXYZ$ into two parts whose areas are in the ratio $7:1$, what is the ratio XA/AY? Express your answer as a common fraction.



5. If the points $(0,0), (2,3)$, and $(4,2)$ are three vertices of a parallelogram, what is the sum of all the possible x-coordinates of the fourth vertex?

6. Express the following sum as a common fraction.

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{9\cdot 10}$$

7. If x is a positive real number such that $x+\frac{1}{x}=4$, what is the value of $x^3+\frac{1}{x^3}$?

8. Four rooks are placed at random on distinct squares on a chessboard. Find the probability that no two rooks can attack each other in their current position. (Note: A chessboard is an eight by eight array of squares. A rook can attack any piece that is on the same row or column it occupies.)

9. If x and y are integers such that $xy - 2005 = x + y$, what is the largest possible value of $x + y$?

10. For each positive integer n, define $p(n)$ to be the sum of the base-ten digits of n. For each positive integer n, define

$$a_n = \begin{cases} p(n) - p(n+1) & \text{if } p(n) - p(n+1) > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Compute $a_1 + a_2 + a_3 + \cdots a_{2006}$.